



# **Optimal Research and Teaching Targets in Academia**

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# Optimal Research and Teaching Targets in Academia\*

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## Abstract

This paper presents a model of optimal research and teaching targets in academia building on the theory of reference-dependent preferences by Tversky and Kahneman (1991) and Koszegi and Rabin (2006). Heterogeneous motivated academics respond to targets decided by a risk-neutral manager whose objective is to maximize the overall teaching and research output. Complementarity and substitutability between the two activities, the academics' productivity, and the department's composition are crucial determinants in setting the targets optimally and they may lead to diversification or specialization in production. The consequences on academics' welfare are also investigated. Several extensions are considered including the possibility of personalized targets and labor mobility.

JEL Codes: D20, I23, J20.

## 1 Introduction

A classical problem in contemporary academia is how to control that academics are working suitably with a right combination of research and teaching activities. This challenging task is further complicated by the fact that universities should guarantee and promote academic freedom. For many years the tenure track system (or probation periods) seemed to be a right compromise between the incentives to work and the protection of academic freedom. In a beautiful satirical note published in 1947, Stigler argued that the system is far from being perfect, but he warned that more sophisticated incentives can create distortions and spoil the balance between research and teaching. Despite Stigler's advice, many universities are now introducing new mechanisms to influence the work of academics.<sup>1</sup>

One of these new approaches consists in fixing research and teaching targets. Very often the head of department (henceforth manager) wants faculty members to reach determined levels of research and/or teaching outcomes coherent with his objectives. The use of targets in universities is achieved in many different variants whose extremes can be described as follows. On one hand, targets are implicitly set by the manager who can pressure

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<sup>1</sup>Ehrenberg (2012) shows that in U.S. the percentage of full-time faculty not on tenure track has more than doubled between 1975 and 2007, increasing from 18.6 percent to 37.2 percent. He also discusses the possible reasons behind this phenomenon.

academics to reach them. For instance, when faculty members do not satisfy such goals, the manager can fix weekly compulsory meeting to monitor their work. On the other hand, there are departments in which targets are explicitly set and a formal system of reward and punishment is in force. For instance, in many UK departments there are targets on research and teaching that should be met by all faculty members. When such goals are not met, the academic needs to undertake some managerial procedure (of questionably usefulness) often called “Performance Improvement Regulations” or “Managing Under-Performance”. As it clearly appears, setting targets in an optimal way is a very challenging task because managers face heterogeneous faculty members and they are often bound by equality regulations or labor law to set the same goals for everybody. Given these difficulties, targets are often chosen arbitrarily and different institutions follow different approaches: in some departments they are equal to the average performances of academics while in others they correspond to a minimum level of performance which is considered sufficient. However there is no contribution, to the best of our knowledge, attempting to tackle the issue of optimal targeting in the academic environment.

The aim of this paper is to fill this gap in the literature and study how a manager should set optimally the targets on research and teaching in academia. We do so by developing a parsimonious model of a university department with one manager and two types of academics: one more productive in research and another more productive in teaching. Each academic has an endowment of effort and he uses it to produce research and teaching. We assume that academics are motivated agents whose objective is represented by constant elasticity of substitution utility function defined over research and teaching.<sup>2</sup> Differently, the manager is risk neutral and aims to maximize the value of the overall amount of research and teaching produced. We interpret the “prices” of research and teaching as external incentives which are exogenously given. To influence the academics’ activities, the manager sets targets on research and teaching which are modeled by using reference points as it is common in the literature (see, for instance, Koszegi and Rabin, 2006).<sup>3</sup> By following this approach, academics’ utility functions satisfy loss aversion (see, for instance, Tversky and Kahneman, 1991) and this implies that performances below the targets loom larger than corresponding performance above them. Academics’ optimal effort allocation between research and teaching is then influenced by the choice of different targets throughout the channel of loss aversion. Furthermore, it is worth to underline that, to the best of our knowledge, we are the first to consider a framework where the targets/reference points are neither determined by the status quo nor by the agent himself, but rather they are set by an external agent, the manager (for a more detailed discussion see Maltz, 2020).

The main result can be summarized as follow. If the external incentives on research and teaching are sufficiently skewed towards one of the two activities, the manager maximizes his payoff by setting targets such that the academics fully specialize in that task. This is not surprising: if the external incentives are much higher for one activity, it is optimal for the manager that all faculty members produce just that activity. More interestingly, when the external incentives on research and teaching are similar, the optimal targets depend also on the composition of the department as well as the academics’ elasticity of substitution. In this case, we show that the manager still sets targets to induce the faculty members to fully specialize in just one activity if research and teaching are complements (low elasticity

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<sup>2</sup>Becker (1975) proposes a model to study the work of academics that is somehow related to ours. He considers academics characterized by utility functions depending on research, teaching, and consumption and by linear technologies to produce research and teaching.

<sup>3</sup>There is an interesting strand of literature which focuses on the problem of New York City cab drivers who self-impose targets on work hours and income. See Farber (2008) and Crawford and Meng (2011).

of substitution) while he sets them to allow diversification if research and teaching are substitutes (high elasticity of substitution) and the shares of the two types of academics are sufficiently close. Under diversification the targets are set in a way that it is optimal for faculty members to produce both research and teaching.

Interestingly, Hattie and Marsh (1996) and Epstein and Menis (2013) study the relationship between research and teaching from an empirical perspective and, despite the limitations of their analysis, they suggest that the two activities are substitutes in the fields of natural sciences while in humanities and social sciences they are more often complements.<sup>4</sup> Therefore our results suggest that the optimal targets are disciplines dependent and that there is scope of diversification only in fields with substitutability between research and teaching when the shares of the two types of academics are close enough.

In the second part of the paper we study some extensions of the model to gain some additional policy insights on using targets in universities. We begin by investigating their effects on academics' welfare because this is an aspect that is often claimed to be key in the universities strategic plans.<sup>5</sup> Our analysis is based on comparing the academics' utilities in the case of optimal targets and in the case of no targets which we evocatively call academic freedom. We show that the results depend on a parameter of the model representing the individual academics' engagement with the targets. More specifically, when faculty members tend to feel gratified by reaching set goals, they may have a higher utility at the optimal targets. On the contrary, if they are not engaged with the targets, they are usually better off at academic freedom, independently of their productivities in research and teaching.

We next study the case of personalized targets where a manager can set different targets for each type of academic. In this extension we scrutinize if there are reasons, different from equality regulations, which discourage the use of personalized targets. Our example shows that the manager is better off by switching to personalized targets as he can fully specialize academics in their most productive activity; on the other hand, one type of academics is always worsen off but for very high level of engagement with the target. Therefore, moving to personalized targets could be a controversial policy considering the negative impact on one group of academics.

In the third extension we take a broader view and investigate the effects of targets on the overall university system. In particular, we study if the use of targets may lead all departments to fully specialize in just one activity by allowing for labor mobility across departments. Our analysis, based on an example with two departments having different shares of academics' types, shows that this is not necessarily the case when academics are not sufficiently engaged with the targets. In other words, we show that diversification is not a specific feature of a close environment but it can still emerge when labor mobility is allowed.

Finally, in the last extension we relax the assumption that academics' effort is costless and we consider a manager that faces the participation constraints when maximizing his

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<sup>4</sup>Hattie and Marsh (1996) states that "A zero relationship [between research and teaching] is typically found (a) in the natural sciences [...]. The scenarios in which the correlation between teaching and research is greater than zero are (a) in social science departments [...]". Epstein and Menis (2013) state that "There are departments where increasing the number of students seems to increase the number of publications per capita: Department of Integrated Life Sciences, History of the Middle East, Israel Studies, Information Studies, Hebrew Language, Mathematics, History of Israel and French Culture." See also De Fraja and Valbonesi (2012) for a discussion of this point in a theoretical paper.

<sup>5</sup>Many universities conduct staff survey to try to improve the academics' working conditions. In the UK, the University and College Union (UCU) conducted a national survey on the stress and well-being among staff in higher education in 2013.

payoff. This allows us to study the interaction between targets and the academics' supply of effort. We find that the participation constraints are usually binding but their impact changes according to academics' engagement with the targets. In other words, when faculty members feel highly gratified by reaching set goals, the optimal manager's choice is very close to the optimal targets without participation constraints.

In conclusion, our overall analysis points out that setting optimal targets is department-specific and a university-wide approach may not be optimal. Furthermore, set goals may have positive or negative effects on the welfare of academics depending on their personal engagement with them. We believe that this is an important aspect to consider in designing an incentive system given the peculiar features of the academics' work. Finally, the extensions of the model do not only give some additional policy insights on the effects of using targets, but they also show that our theoretical model is quite flexible and can be adapted to study more specific problems.

The paper is organized as follows. In Section 2, we introduce the model. In Section 3, we state our main results. In Section 4, we consider some examples and extensions of the model. In Section 5, we draw some conclusions. All the proofs can be found in the Appendix.

## 2 Model Setup

### 2.1 Manager and academics

Let us consider a university department with  $N$  academics and 1 manager.

We first describe the academics. They are motivated agents having utilities depending on research  $x$  and teaching  $y$

$$u(x, y) = \left( \frac{1}{2}x^\beta + \frac{1}{2}y^\beta \right)^{\frac{1}{\beta}}, \quad (1)$$

with  $\beta \in (-\infty, 1)$ . This is a constant elasticity of substitution (CES) utility function with elasticity  $\epsilon = \frac{1}{1-\beta}$ . Note that research and teaching can be seen as complement if  $\epsilon \in (0, 1)$  and as substitute if  $\epsilon \in (1, \infty)$ . In our analysis we assume that all academics in the department have the same elasticity. Each academic holds an amount of effort  $E$  which can be used to produce research and teaching according to the following linear production functions

$$x(e) = m \cdot e \quad \text{and} \quad y(e) = n \cdot (E - e),$$

where  $e \in [0, E]$  and  $(E - e)$  denote the amount of effort devoted to research and teaching respectively and  $m, n > 0$ . We focus our analysis on the case in which all effort is used in the production activities.<sup>6</sup>

In the departments there are two types of academics who have different productivities. Academics of type 1 are more productive in research  $m_1 > n_1$  while academics of type 2 are more productive in teaching  $n_2 > m_2$ . We denote by  $\pi$  the share of academics of type 1 and, consequently,  $1 - \pi$  is the share of academics of type 2. Note that  $\pi$  has the key role of describing the composition of the department. Furthermore,  $x_i$  and  $y_i$  denote the amount of research and teaching produced by an academic of type  $i$ .

We next describe the manager. We assume that the manager is risk neutral and knows

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<sup>6</sup>A similar approach is followed by Holmstrom and Milgrom (1991) who explain that "a worker on the job may take pleasure in working up to some limit". Such a limit can be assumed to be  $E$  in our analysis.

the academics' productivities and the composition of the department  $\pi$ . His payoff function is then

$$V(x_1, y_1, x_2, y_2) = N(\pi(p_x x_1 + p_y y_1) + (1 - \pi)(p_x x_2 + p_y y_2)). \quad (2)$$

The parameters  $p_x$  and  $p_y$  are the external incentives on research and teaching that the manager receives exogenously and we assume that  $p_x, p_y > 0$ . In a public university system we may think that when the government wants to increase the output of research, it sets a higher  $p_x$ , the government funding per unit of research.<sup>7</sup> Differently in a private system, it may be the central administration of the university that decides  $p_x$  and  $p_y$ .<sup>8</sup> Given the values of the parameters  $p_x$  and  $p_y$ , the manager aims to maximize the payoff function by influencing the academics' allocation of effort between research and teaching. Obviously, there are many mechanisms and incentive systems that the manager can use to do that (e.g. career promotions, financial incentives, and non-monetary prizes). In this paper we focus on the non-pecuniary tool of setting targets over research and teaching. In particular we consider the case in which the manager fixes the same targets for all academics. This is often the case in order to avoid discrimination among academics working under the same contract. There can also be some external equality regulations, imposed by labor law or agreements with unions, which forbid the use of personalized targets.

We next explain how we model targets in the paper.

## 2.2 Reference-dependent preferences and targets

Our modeling approach follows the strand of literature which uses reference points to represent targets (see Koszegi and Rabin, 2006; Farber, 2008; Crawford and Meng, 2011). We then introduce in the utility function (1) the reference points  $r_x$  and  $r_y$  in the following way

$$u(x, y; r_x, r_y) = A(r_x, r_y) \left( \frac{1}{2} r_x^{\rho - \beta} x^\beta + \frac{1}{2} r_y^{\rho - \beta} y^\beta \right)^{\frac{1}{\beta}}, \quad (3)$$

with  $-\infty < \beta < \rho < 1$ ,  $r_x, r_y \geq 0$ , and  $A(r_x, r_y)$  is an arbitrary normalization represented by a positive continuous function. This utility function, which satisfies loss aversion, was proposed by Munro and Sugden (2003) and similar functional forms are also used in macroeconomic papers on habits as Ravn et al. (2006).<sup>9</sup> In our model the reference points  $r_x$  and  $r_y$  represent the targets set by the manager on research and teaching respectively. Note that the normalization rule from the perspective of the academics is simply a constant because  $r_x$  and  $r_y$  are chosen by the manager. Finally, the parameter  $\rho$  represents the academics' level of engagement with the targets. Its role will be further discussed in the next sections.

## 3 Optimal behaviors

In our model we assume that the manager moves first and sets the targets to maximize his payoff. Afterwards academics choose their optimal allocation of effort between research and teaching by maximizing the utility function in (3). We solve the model by backward

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<sup>7</sup>A classical example of government funding based on the research output is the Research Excellent Framework (REF) in United Kingdom.

<sup>8</sup>In both cases, the external incentives can also be modified by the policy makers in response to exogenous shocks affecting the entire academic sector.

<sup>9</sup>The specification of the utility function in Section 4.2 of Ravn et al. (2006) becomes equivalent to ours when there are just two goods and their parameter  $\theta$  is equal to  $1 - \frac{\rho}{\beta}$ .

induction and we then start by studying the academics' optimal behavior.

### 3.1 Academics' optimal behavior

Consider an academic of type  $i$ . His maximization problem can be written as

$$\begin{aligned} \max_{e_i} \quad & A(r_x, r_y) \left( \frac{1}{2} r_x^{\rho-\beta} x_i(e_i)^\beta + \frac{1}{2} r_y^{\rho-\beta} y_i(e_i)^\beta \right)^{\frac{1}{\beta}}, \\ \text{s.t.} \quad & e_i \in [0, E]. \end{aligned} \quad (4)$$

with  $x_i(e_i) = m_i e_i$  and  $y_i(e_i) = n_i(E - e_i)$ . By solving the maximization problem (see Proposition 1 in the Appendix), we find that the optimal effort on research for an academic of type  $i$  is

$$e_i^* = \frac{E}{1 + \left(\frac{m_i}{n_i}\right)^{\frac{\beta}{\beta-1}} \left(\frac{r_x}{r_y}\right)^{\frac{\rho-\beta}{\beta-1}}}. \quad (5)$$

Remember that the optimal effort of teaching is given by the residual  $E - e_i^*$ .

An important point to highlight is that the optimal effort  $e_i^*$  depends on the ratio of the targets. For this reason, the rest of our analysis will be developed in terms of the target ratio  $\theta = \frac{r_x}{r_y}$ . It is then convenient to define the function  $e_i^*(\theta)$  which associates to  $\theta$  the optimal effort  $e_i^*$  as follows

$$e_i^*(\theta) = \frac{E}{1 + a_i \theta^b}, \quad (6)$$

with  $a_i = \left(\frac{m_i}{n_i}\right)^{\frac{\beta}{\beta-1}} > 0$  and  $b = \frac{\rho-\beta}{\beta-1} < -1$ . We discuss some properties of this function in Proposition 1. Given these results, we can now move to study the manager's optimal choice of targets.

### 3.2 Manager's optimal targets (ratio)

In departments using targets, the manager usually sets a target on research and another on teaching. Nonetheless, our results on the academics' behaviors suggest that what actually matters is the target ratio  $\theta = \frac{r_x}{r_y}$ . For this reason, in our analytical analysis on the use of targets in academia we consider  $\theta$  as the strategic variable of the manager.<sup>10</sup>

The interpretation of the variable  $\theta$  can be the following. When the manager wants to increase the output of research, he sets  $\theta > 1$ , which implies  $r_x > r_y$ , to push all academics to allocate more effort on research. Differently, when the manager wants to prioritize teaching over research, he sets  $\theta < 1$  that implies  $r_y > r_x$ . In the former we say that targets are skewed toward research while in the latter we say that targets are skewed toward teaching. The case  $\theta = 1$  means that neither research nor teaching are prioritized by the manager. Quite interestingly, in such a case the optimal effort  $e_i^*(1)$  is the solution to the maximization problem with both the utility function without targets in (1) and the utility function with targets in (3). Heuristically, when  $\theta = 1$  it is as if the manager leaves the academics free to maximize their utility function without targets. For this reason, we refer to this case as academic freedom.<sup>11</sup>

<sup>10</sup>In any case the reader can still think that the manager sets  $r_x$  and  $r_y$  by assuming that he is using a normalization rule which associates to any  $\theta$  a unique pair  $(r_x, r_y)$ . It is worth reminding that a similar situation arise also in general equilibrium where a normalization rule is required to find the vector of competitive prices.

<sup>11</sup>Generally speaking academic freedom means freedom to research, to teach or to communicate ideas and facts without any limitation posed by the authority. Nevertheless, academic freedom may also include the

By considering the function  $e_i^*(\theta)$  and the academics' production functions, let us define  $x_i(\theta) = x_i(e_i(\theta))$  and  $y_i(\theta) = y_i(e_i(\theta))$ , for  $i = 1, 2$ . Therefore, the manager's payoff in (2) is actually a function of  $\theta$ , i.e.,  $V(\theta)$ . Furthermore, since  $r_x, r_y \geq 0$ , it follows that  $\theta$  lies in the set  $[0, \infty)$ . From these results, the manager's maximization problem can be written as<sup>12</sup>

$$\begin{aligned} \sup_{\theta} \quad & N(\pi(p_x x_1(\theta) + p_y y_1(\theta)) + (1 - \pi)(p_x x_2(\theta) + p_y y_2(\theta))), \\ \text{s.t.} \quad & \theta \in [0, \infty). \end{aligned} \tag{7}$$

Furthermore, we interpret  $\theta = 0$  as the case in which the manager is not interested in research (i.e.,  $r_x = 0$ ) and  $\theta \rightarrow \infty$  as the case in which he is not interested in teaching (i.e.,  $r_y = 0$ ). We denote by  $\theta^*$  the optimal targets ratio that solves (7). Henceforth, we refer to  $\theta^*$  simply as the optimal targets.

To study the solutions of the maximization problem we need to consider three different cases

- $\frac{p_x}{p_y} \in (0, \frac{n_1}{m_1}]$ , i.e., external incentives skewed toward teaching.
- $\frac{p_x}{p_y} \in [\frac{n_2}{m_2}, \infty)$ , i.e., external incentives skewed toward research.
- $\frac{p_x}{p_y} \in (\frac{n_1}{m_1}, \frac{n_2}{m_2})$ , i.e., balanced external incentives.

The next two theorems show what are the optimal targets in the case of skewed external incentives and balanced external incentives respectively.

**Theorem 1.** Let  $\frac{p_x}{p_y} \notin (\frac{n_1}{m_1}, \frac{n_2}{m_2})$ . Then the manager has the following optimal behaviors:

- Full specialization in teaching when external incentives are skewed toward teaching. Formally,  $\theta^* = 0$  when  $\frac{p_x}{p_y} \in (0, \frac{n_1}{m_1}]$ .
- Full specialization in research when external incentives are skewed toward research. Formally,  $\theta^* \rightarrow \infty$  when  $\frac{p_x}{p_y} \in [\frac{n_2}{m_2}, \infty)$ .

The intuition of the theorem is immediate. In case (i), the external incentives ratio is lower than the marginal rate of transformation for both types of academics, i.e.,  $\frac{p_x}{p_y} \leq \frac{n_1}{m_1} < \frac{n_2}{m_2}$ . Therefore, the manager maximizes his payoff by pushing all academics to allocate all their effort on teaching. He does so by setting  $\theta^* = 0$  which implies  $e_i^*(0) = 0$  for  $i = 1, 2$ . For this reason, we say that in this case the optimal targets consist in full specialization in teaching. Differently, in case (ii), the external incentives ratio is higher than the marginal rate of transformation for both types of academics and then the manager sets  $\theta^* \rightarrow \infty$  for which we have that  $\lim_{\theta \rightarrow \infty} e_i^*(\theta) = E$  for  $i = 1, 2$ . In this case we say that the optimal targets consist in full specialization in research.

The case of balanced external incentives is probably the most interesting because the analysis is richer and it depends on the preferences of academics (the elasticity of substitution  $\epsilon = \frac{1}{1-\beta}$ ) and the composition of department, represented by  $\pi$ .

To develop our analysis for balanced external incentives we define the following thresholds:

$$\tilde{\pi}_1 = \frac{(\tilde{n}_2 - \tilde{m}_2)a_1}{(\tilde{n}_2 - \tilde{m}_2)a_1 + (\tilde{m}_1 - \tilde{n}_1)a_2}, \quad \tilde{\pi}_2 = \frac{\tilde{n}_2 - \tilde{m}_2}{\tilde{n}_2 - \tilde{m}_2 + \tilde{m}_1 - \tilde{n}_1}, \quad \tilde{\pi}_3 = \frac{(\tilde{n}_2 - \tilde{m}_2)a_2}{(\tilde{n}_2 - \tilde{m}_2)a_2 + (\tilde{m}_1 - \tilde{n}_1)a_1},$$

freedom of choosing different teaching approaches which may require different allocation of effort between research and teaching. See Russel (1957) for an interesting discussion on academic freedom.

<sup>12</sup>We consider the supremum because the constrained set is open.

where, to simplify the notation, we have that  $\tilde{m}_i = p_x m_i$  and  $\tilde{n}_i = p_y n_i$ . The following relationships hold (see Lemma 1 in the Appendix)

- $0 < \tilde{\pi}_3 \leq \tilde{\pi}_2 \leq \tilde{\pi}_1 < 1$  when  $\epsilon \in (0, 1]$ .
- $0 < \tilde{\pi}_1 < \tilde{\pi}_2 < \tilde{\pi}_3 < 1$  when  $\epsilon \in (1, \infty)$ .

These thresholds depend on the productivities, the external incentives, and the elasticity of substitution, and they have a key role in determining the manager's optimal choice.

The next theorem proves what is the manager's optimal choice in the case of balanced external incentives.

**Theorem 2.** Let  $\frac{p_x}{p_y} \in (\frac{n_1}{m_1}, \frac{n_2}{m_2})$ . Then the manager has the following optimal behaviors:

- Assume low elasticity, i.e.,  $\epsilon \in (0, 1]$ .
  - Full specialization in teaching when the share of academics being more productive in research is sufficiently low. Formally,  $\theta^* = 0$  when  $\pi \in [0, \tilde{\pi}_2]$ .
  - Full specialization in research when the share of academics being more productive in research is sufficiently high. Formally,  $\theta^* \rightarrow \infty$  when  $\pi \in [\tilde{\pi}_2, 1]$ .<sup>13</sup>
- Assume high elasticity, i.e.,  $\epsilon \in (1, \infty)$ .
  - Full specialization in teaching when the share of academics being more productive in research is sufficiently low. Formally,  $\theta^* = 0$  when  $\pi \in [0, \tilde{\pi}_1]$ .
  - Diversification in research and teaching when the share of academics being more productive in research and teaching are sufficiently close. Formally,

$$\theta^* = \left( \frac{-a_2 \pi k_1 - a_1 (1 - \pi) k_2 + (a_1 - a_2) \sqrt{-\pi k_1 (1 - \pi) k_2}}{a_2^2 \pi k_1 + a_1^2 (1 - \pi) k_2} \right)^{\frac{1}{b}}, \quad (8)$$

with  $k_1 = a_1(\tilde{m}_1 - \tilde{n}_1)$  and  $k_2 = a_2(\tilde{m}_2 - \tilde{n}_2)$ , when  $\pi \in (\tilde{\pi}_1, \tilde{\pi}_3)$ .

- Full specialization in research when the share of academics being more productive in research is sufficiently high. Formally,  $\theta^* \rightarrow \infty$  when  $\pi \in [\tilde{\pi}_3, 1]$ .

An important insight of the theorem is that when research and teaching are complements ( $\epsilon \in (0, 1]$ ) the optimal targets always consist in full specialization. Heuristically, the intuition is the following. When research and teaching are complements, the use of targets is less effective because academics are less willing to reallocate large amount of effort between the two activities. Therefore, the manager needs to set  $\theta^*$  at the limit of the constrained region ( $\theta^* = 0$  or  $\theta^* \rightarrow \infty$ ) in order to influence the effort's choices of academics. Differently, when research and teaching are substitutes ( $\epsilon \in (1, \infty)$ ) targets are more compelling in influencing the allocation of effort between research and teaching and, therefore, a new solution emerges which is diversification in research and teaching. In such a case, the manager finds optimal that academics allocate efforts on both research and teaching. Finally, it is worth noting the key role of the department's composition  $\pi$  in determining the optimal targets.

When consider the case of diversification some questions arise: Are the optimal targets skewed toward research ( $\theta^* > 1$ ) or toward teaching ( $\theta^* < 1$ )? Can academic freedom be an optimal choice for the manager ( $\theta^* = 1$ )? The next corollary addresses such issues.

<sup>13</sup>When  $\pi = \tilde{\pi}_2$ , the manager's payoff is the same for the full specialization in research and the full specialization in teaching.

**Corollary 1.** Let  $\frac{p_x}{p_y} \in (\frac{n_1}{m_1}, \frac{n_2}{m_2})$ ,  $\epsilon \in (1, \infty)$  and  $\pi \in (\tilde{\pi}_1, \tilde{\pi}_3)$ . Furthermore, let's define the following threshold

$$\bar{\pi} = \frac{a_2(\tilde{n}_2 - \tilde{m}_2)(a_1 + 1)^2}{a_2(\tilde{n}_2 - \tilde{m}_2)(a_1 + 1)^2 + a_1(\tilde{m}_1 - \tilde{n}_1)(a_2 + 1)^2}.$$

Then, it follows that  $\bar{\pi} \in (\tilde{\pi}_1, \tilde{\pi}_3)$  and the manager's optimal behaviors are:

- Diversification skewed toward teaching,  $\theta^* < 1$ , when  $\pi \in (\tilde{\pi}_1, \bar{\pi})$ .
- Academic freedom,  $\theta^* = 1$ , when  $\pi = \bar{\pi}$ .
- Diversification skewed toward research,  $\theta^* > 1$ , when  $\pi \in (\bar{\pi}, \tilde{\pi}_3)$ .

Also in this case, we can see that the composition of department  $\pi$  has a key role in determining the kind of diversification that there is at the optimal targets. Furthermore, the corollary shows that there is only one value of  $\pi$  for which  $\theta^* = 1$  and then we can say that generically academic freedom is not an optimal choice for the manager.

The results in Theorems 1 and 2 and Corollary 1 can help to explain why universities occupy different positions in rankings based on research and based on teaching. For example, in a ranking based on research, universities fully specialized in research will be at the top while at the bottom we have universities fully specialized in teaching. In addition, universities with a more uniform academic composition tend to appear in the middle of these rankings.<sup>14</sup>

We conclude this section by analyzing the role of  $\rho$  which is the parameter that characterizes the academics' level of engagement with the target. First, we note that although  $\rho$  affects  $\theta^*$ , it has no impact on the optimal efforts of academics. By substituting  $\theta^*$  given by (8) in the formula of the optimal effort (6), it is immediate to see that  $e_i^*(\theta^*)$  does not depend on  $\rho$ . Also in the case of full specialization, when  $\theta^* = 0$  or  $\theta^* \rightarrow \infty$ , the parameter  $\rho$  does not have any role in determining the academics' optimal efforts. We investigate more formally the effect of  $\rho$  on the academics' optimal choices in Proposition 2 in the Appendix.

One of the main effect of  $\rho$  is on the optimal targets  $\theta^*$  as the following corollary clarifies.

**Corollary 2.** Let  $\frac{p_x}{p_y} \in (\frac{n_1}{m_1}, \frac{n_2}{m_2})$ ,  $\epsilon \in (1, \infty)$ , and  $\pi \in (\tilde{\pi}_1, \tilde{\pi}_3)$ . Let  $\theta^*(\rho)$  be the function that associate to each  $\rho \in (\beta, 1)$  the optimal targets  $\theta^*$  given by (8). Then,

- $\frac{d\theta^*(\rho)}{d\rho} > 0$  when  $\theta^* < 1$ ;
- $\frac{d\theta^*(\rho)}{d\rho} < 0$  when  $\theta^* > 1$ .

The corollary shows that the higher the value of  $\rho$  is, the closer to 1 is  $\theta^*$ . In fact, an increase in  $\rho$  increases the value of  $\theta^*$  when  $\theta^* < 1$  and decreases its value when  $\theta^* > 1$ . This result suggests that  $\rho$  represents the level of academics' engagement with the targets. In other words, when  $\rho \rightarrow 1$ , academics are very engaged with the targets and small variations of  $\theta$  around 1, the case of academic freedom, are sufficient to modify the academics' behaviors in a way that is optimal for the manager. Differently, when  $\rho \rightarrow \beta$ , academics are less engaged with the targets and the manager has to choose a  $\theta^*$  that is further away from 1 to influence academics' behavior effectively. We further discuss the role of  $\rho$  in the next examples.

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<sup>14</sup>The reader may find some evidence of this claim by comparing the position of UK economic departments, for example, in the Guardian University Guide with the REF results.

## 4 Extensions and policy applications

The aim of this section is to show how our framework, despite its simplicity, can provide a useful analytical tool to think about targets in academia. We propose some examples to address different policy questions that may arise when using targets. The examples also show that it is possible to extend our model to study more specific problems.

### 4.1 Effects of optimal targets

The first question that arises when using targets in academia is: “what are the effects of targets on the output and welfare of academics?”

To address this question we compare the optimal effort of academics at academic freedom,  $\theta = 1$ , and at the optimal targets,  $\theta = \theta^*$ . The analysis is based on the following example.

**Example 1.** Academics of type 1 have productivities  $m_1 = 4$  and  $n_1 = 2$  while academics of type 2 have productivities  $m_2 = 1$  and  $n_2 = 4$ . Furthermore, each of them holds one unit of effort,  $E = 1$ , and their elasticity of substitution is  $\epsilon = \frac{5}{4}$  which implies  $\beta = \frac{1}{5}$ . Consider a department where the proportion of academics of type 1 is  $\pi = \frac{12}{25}$  and the manager external incentives are  $p_x = 1$  and  $p_y = 1$ . We then have balanced external incentives as  $\frac{p_x}{p_y} \in (\frac{1}{4}, 4)$  and the probability thresholds are  $\tilde{\pi}_1 = 0.47$ ,  $\bar{\pi} = 0.59$ ,  $\tilde{\pi}_2 = 0.6$ ,  $\tilde{\pi}_3 = 0.72$ . Since  $\pi = 0.48 \in (\tilde{\pi}_1, \tilde{\pi}_3)$ , it is optimal to have diversification in research and teaching by Theorem 2 and, since  $\pi = 0.48 \in (\tilde{\pi}_1, \bar{\pi})$ , the diversification is skewed toward teaching by Corollary 1. Finally,  $\theta^* = e^{\frac{2.634}{0.2-\rho}}$  and the corresponding academics’ optimal efforts are given in Table 1.

	Effort in research	Effort in teaching
Type 1 academics at $\theta = 1$	0.54	0.46
Type 1 academics at $\theta = \theta^*$	0.04	0.96
Type 2 academics at $\theta = 1$	0.41	0.59
Type 2 academics at $\theta = \theta^*$	0.02	0.98

Table 1: Optimal efforts in research and teaching

We first consider the effects of targets on the choices of academics. In the example the manager finds optimal to push all academics to produce more teaching and he uses the targets to achieve such goals. The academics respond to targets skewed toward teaching by increasing the effort in teaching and correspondingly reducing the effort on research as shown by Table 1. Therefore, we can immediately conclude that targets are a useful tool that the manager can use to influence the work of academics. The adjustment of the academics to the optimal targets can be shown graphically. In Figures 1a and 1b we have described how the optimal efforts as well as the indifference curves look like at academic freedom and optimal targets. Academics of type 1 are represented by the blue color and academics of type 2 are represented by the black color. The optimal amounts of research and teaching at academic freedom for an academic of type  $i$  is the point  $(x_i^o, y_i^o)$  and the dashed lines are the corresponding indifference curves. Similarly, the optimal amounts of research and teaching for an academic of type  $i$  at the optimal targets are the points  $(x_i^*, y_i^*)$  and the solid lines are the corresponding indifference curves. Figures 1a and 1b are also useful to visualize the manager’s optimal targets. In fact, the slope of the red line is the inverse of the optimal target ratio. The case of academic freedom corresponds to the 45

degree line while the line is steeper for targets skewed toward teaching and it is flatter for targets skewed toward research. It is interesting to note that for both academics the optimal output ratio  $\frac{x_i(\theta^*)}{y_i(\theta^*)}$  is different from the optimal targets (ratio). This is due to the fact that academics are heterogeneous and the manager is using the same optimal targets for both types of academics. As we will see below, this is not the case when personalized targets are used.

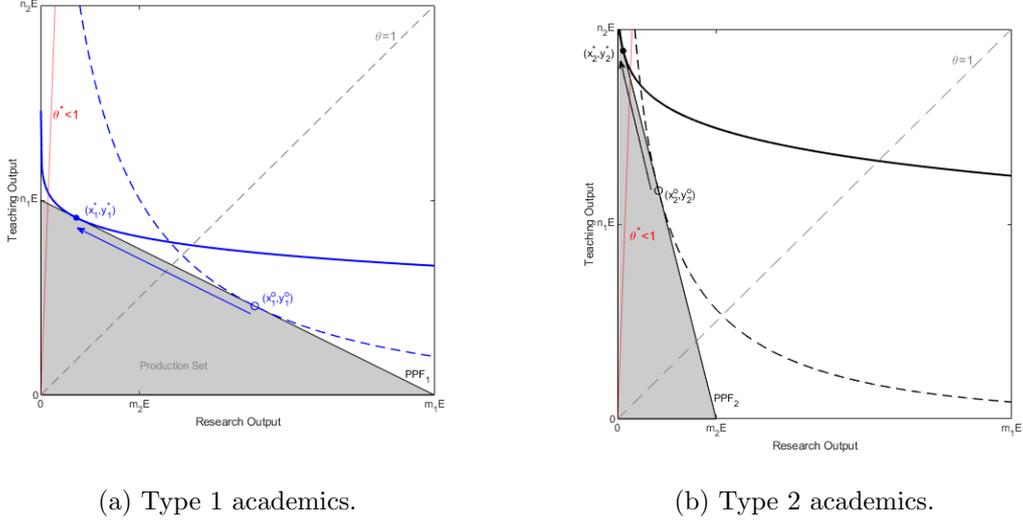


Figure 1: Academic freedom and optimal targets for the two types of academics.

We continue our analysis by studying the welfare of the academics. First, we compare the utilities of the two types of academics in the case of academic freedom and in the case of optimal targets. Consider Figures 2a and 2b. By comparing the indifference curves, we observe that type 1 academics are better off than type 2 academics under academic freedom (Figure 2a). This is not surprising given the configuration of productivities ( $m_1 = n_2$  and  $n_1 > m_2$ ). Differently, type 2 academics are better off than type 1 academics at the optimal targets (Figure 2b). This is due to the fact that targets are skewed toward teaching and then the academics more productive in teaching benefit more from the use of targets than the academics more productive in research ( $n_2 > n_1$ ).

Next, we compare the utilities of academics at academic freedom and at the optimal targets. Looking again at Figures 1a and 1b, it emerges that this is a more challenging task because the indifference curves corresponding to the two cases cross each other and, therefore, we cannot conduct an analysis based on the concept of ordinal utilities.<sup>15</sup> As a consequence, we need to choose a normalization  $A(r_x, r_y)$  in order to calculate the magnitude of academics' utilities. In the rest of our analysis we focus on the one proposed by Munro and Sugden (2003) (henceforth MS normalization)

$$A(r_x, r_y) = \left( \frac{1}{2} r_x^\rho + \frac{1}{2} r_y^\rho \right)^{\frac{1}{\rho} - \frac{1}{\beta}}.$$

This has the main advantage of making the academics' utility functions depending on  $\theta$  which is the strategic variable of the manager. Put it differently, with the MS normalization

<sup>15</sup>Note that the comparison between the magnitudes of the same utility function at two different targets arises the same difficulties as the comparison between the magnitudes of two different utility functions.

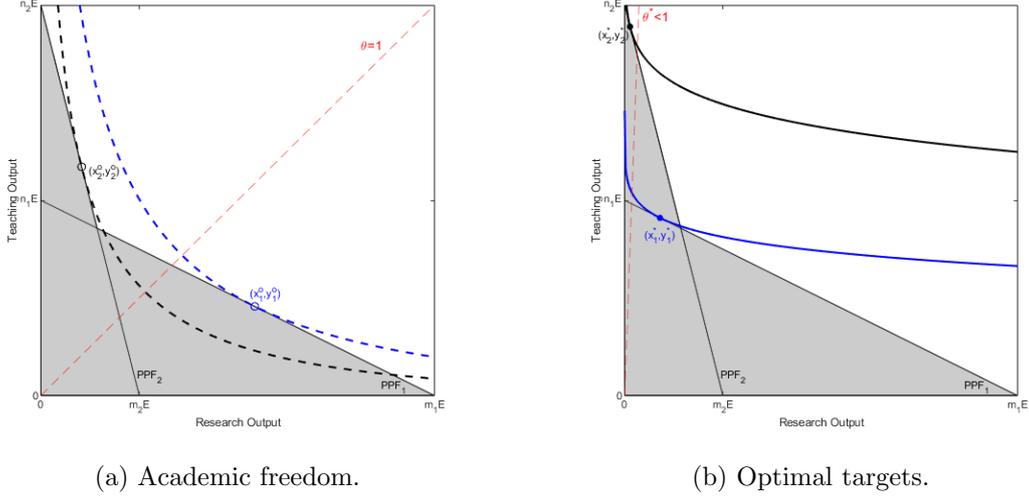


Figure 2: Utility comparison: academic freedom and optimal targets.

rule, the utility function in (3) can be written as

$$u(x, y; \theta) = \left( \frac{1}{2}\theta^\rho + \frac{1}{2} \right)^{\frac{1}{\rho} - \frac{1}{\beta}} \left( \frac{1}{2}\theta^{\rho-\beta}x^\beta + \frac{1}{2}y^\beta \right)^{\frac{1}{\beta}}. \quad (9)$$

Table 2 shows the magnitude of the academics' utilities for different value of  $\rho$  at academic freedom and at the optimal targets. As the table makes clear, the welfare analysis depends

	$\rho =$				
	$\approx \beta$	$2\beta$	$3\beta$	$4\beta$	$\approx 5\beta$
$u_1$ at $\theta = 1$	1.44	1.44	1.44	1.44	1.44
$u_1$ at $\theta = \theta^*$	0.08	0.41	0.70	0.89	1.03
$u_2$ at $\theta = 1$	1.06	1.06	1.06	1.06	1.06
$u_2$ at $\theta = \theta^*$	0.14	0.77	1.31	1.67	1.92

Table 2: Utilities of the two types of academics at academic freedom and at the optimal targets for different values of  $\rho$ .

crucially on  $\rho$ . For  $\rho$  close to  $\beta$ , academics are not engaged with the targets and their use is utility detrimental for both academics. For higher values of  $\rho$  closer to 1, the impact of the targets depends on academics' productivities. Since for the manager is optimal the diversification skewed toward teaching, type 2 academics are better off at the optimal targets because they are more productive in teaching while type 1 academics are always better off at academic freedom because they are more productive in research. We can formalize such insights by studying the limits for  $\rho \rightarrow \beta$  and  $\rho \rightarrow 1$  of the academics' utility functions at the optimal targets. Note that  $x_i(\theta^*)$  and  $y_i(\theta^*)$  do not depend on  $\rho$  as pointed out in Subsection 3.3. We then have

$$\lim_{\rho \rightarrow \beta} u_i(x_i(\theta^*), y_i(\theta^*); \theta^*) = \left( \frac{1}{2}x_i(\theta^*)^\beta + \frac{1}{2}y_i(\theta^*)^\beta \right)^{\frac{1}{\beta}} \quad (10)$$

$$\lim_{\rho \rightarrow 1} u_i(x_i(\theta^*), y_i(\theta^*); \theta^*) = \left( \frac{1}{2}\theta^{*\rho} + \frac{1}{2} \right)^{1 - \frac{1}{\beta}} \left( \frac{1}{2}\theta^{*1-\beta}x_i(\theta^*)^\beta + \frac{1}{2}y_i(\theta^*)^\beta \right)^{\frac{1}{\beta}} \quad (11)$$

Equation (10) shows that when  $\rho \rightarrow \beta$  the utility function corresponds to the case of no targets in (1) and, therefore, any deviation from the optimal effort allocation at academic freedom,  $(x_i(1), y_i(1))$ , is utility detrimental as Table 2 confirms. Differently, Equation (11) shows that when  $\rho \rightarrow 1$  targets have a greater impact on the academics' utilities and their effects may be positive or negative depending on academics' productivities as shown in Table 2.

Despite the fact that our analysis is based on a particular example, an important conclusion may be drawn: the use of targets can greatly change the well-beings of the different groups of academics.

## 4.2 Personalized targets

The next question that we try to address is: “are personalized targets a better tool to influence the work of academics?”.

So far we have always considered a manager constrained to use the same targets for all academics which is the most common situation often due to equality regulations. Only in this section we call such targets “common targets”. Nonetheless it is worth to investigate what may happen if the manager is allowed to use “personalized targets” i.e., different targets for different types of academics. In particular, we try to investigate if their use can improve the welfare of both the manager and the academics.

Formally, when personalized targets are allowed, the manager can choose a  $\theta_1$ , the targets for type 1 academics, and a  $\theta_2$ , the targets for type 2 academics. The manager's maximization problem becomes then

$$\begin{aligned} \sup_{\theta_1, \theta_2} \quad & N(\pi(p_x x_1(\theta_1) + p_y y_1(\theta_1)) + (1 - \pi)(p_x x_2(\theta_2) + p_y y_2(\theta_2))), \\ \text{s.t.} \quad & \theta_1 \in [0, \infty), \\ & \theta_2 \in [0, \infty). \end{aligned} \tag{12}$$

The following theorem shows what are the  $\theta_1^*$  and  $\theta_2^*$  that solve the maximization problem.

**Theorem 3.** Let  $\frac{p_x}{p_y} \in (\frac{n_1}{m_1}, \frac{n_2}{m_2})$ . The manager sets the following optimal personalized targets:  $\theta_1^* \rightarrow \infty$  and  $\theta_2^* = 0$ .

Not surprisingly, the theorem shows that under balanced external incentives academics of type 1 fully specialize in research and academics of type 2 fully specialize in teaching. In other words, with personalized targets, the manager pushes the academics to allocate all the effort in the activity in which they are more productive. Note also that this result depends neither on the academics' elasticity of substitution nor on the composition of the department  $\pi$ . Furthermore, by considering the department described in Example 1, it is possible to show that the manager is better off with personalized targets: his total payoff increases by 32% when compared with the common targets case (from 302.3 to 400). Observe also that this does not mean that both research and teaching outputs are increased; in fact research output increases by 859% (from 20.5 to 196) while teaching reduces by -27% (from 281.9 to 204).

By continuing to focus on Example 1 we now consider the effects that personalized targets have on the welfare of academics. By considering the MS normalization, we can easily verify that the academics' utilities under the optimal personalized targets become

$$\lim_{\theta_1^* \rightarrow \infty} u_1(x_1(\theta_1^*), y_1(\theta_1^*); \theta_1^*) = \left(\frac{1}{2}\right)^{\frac{1}{\rho}} m_1 E \quad \text{and} \quad u_2(x_2(0), y_2(0); 0) = \left(\frac{1}{2}\right)^{\frac{1}{\rho}} n_2 E.$$

Table 3 reports the magnitudes of academics' utilities for different values of  $\rho$  at academic freedom, optimal common targets, and optimal personalized targets. As above, the table

	$\rho =$				
	$\approx \beta$	$2\beta$	$3\beta$	$4\beta$	$\approx 5\beta$
$u_1$ at $\theta = 1$	1.44	1.44	1.44	1.44	1.44
$u_1$ at $\theta = \theta^*$	0.08	0.41	0.70	0.89	1.03
$u_1$ at $\theta_1^* \rightarrow \infty$	0.13	0.71	1.26	1.68	2.00
$u_2$ at $\theta = 1$	1.06	1.06	1.06	1.06	1.06
$u_2$ at $\theta = \theta^*$	0.14	0.77	1.31	1.67	1.92
$u_2$ at $\theta_2^* = 0$	0.13	0.71	1.26	1.68	2.00

Table 3: Utilities of the two types of academics at academic freedom, at the optimal targets, and the the personalized targets for different values for  $\rho$ .

shows that the welfare analysis depends on  $\rho$ . Note also that at the optimal personalized targets academics utilities are the same for both types because their productivities on the activities in which they specialize are equal. The table shows that for  $\rho$  close to  $\beta$ , both types of academics are better off at academic freedom. Again this is not surprising when observing (10). More interestingly, when comparing common targets and personalized targets we find different results for the two types of academics. On one hand, type 2 academics are better off with common targets. This can be explained by the fact that CES utility function displays love for variety and then academics have some utility gains when they produce both research and teaching. Heuristically, academics of type 2 gain more with common targets because they place effort on both the activities and prioritize the one with the highest productivity. On the other hand, type 1 academics are better off with personalized targets. Heuristically, for type 1 academics the gains deriving from producing both research and teaching are offset by the fact that targets are skewed toward teaching, the activity with lower productivity. Put it differently, the full specialization in research gives an higher utility to type 1 academics because the gains of placing all the effort in the most productive activity more than compensate the losses deriving from producing only one good.

Finally, for higher values of  $\rho$  both types of academics are better off with personalized targets. This is not surprising because in such a case academics are very engaged with the targets and, consequently, the gains of producing only their most productive activity compensate the losses of producing only one good.

Our example points out that the effects of personalized targets is positive for the manager while depends on  $\rho$  for the academics. In particular, we show that cases in which the use of personalized targets is utility detrimental for one type of academics easily arise. For this reason, moving from common targets to personalized targets cannot be seen, in general, as a Pareto improvement.

### 4.3 Mobility and departments' specialization

We next investigate if the use of targets leads departments to specialize in research or teaching. That is: "does the mobility of academics among departments lead to departments specializing in just one activity?"

To deal with this problem we modify the original model by considering two departments A and B and allowing academics to move between them. In this modified model, each

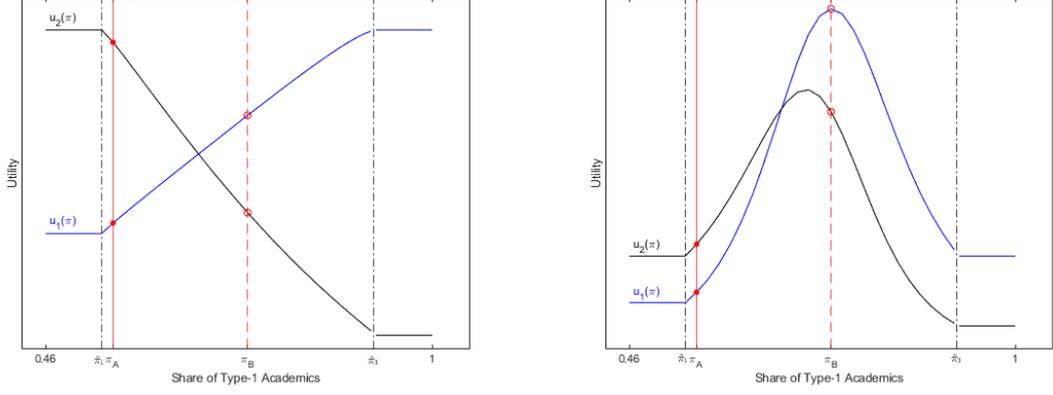
academic is characterized by his type and by the department to which he belongs. We assume that a pair of different types of academics can swap department if both of them increase their utilities by moving. For example, an academic can move from department A to B if his utility increases and there is an academic of different type in B that increases his utility by moving to A. The analysis is based on the following two examples.

**Example 2.** Consider the types of academics given in Example 1 and let  $\rho = \frac{9}{10}$ . We also adopt the MS normalization rule. Consider a department A with 25 academics, 12 of type 1 and 13 of type 2, i.e.,  $\pi_A = \frac{12}{25}$ . Consider a department B with 25 academics, 15 of type 1 and 10 of type 2, i.e.,  $\pi_B = \frac{15}{25}$ . The managers external incentives are  $p_x = 1$  and  $p_y = 1$  in both departments. As the probability thresholds are the same of Example 1, by Corollary 1 it follows that the optimal targets consist in diversification skewed toward teaching in department A and diversification skewed toward research in department B. We consider Figure 3a to study academics mobility. The blue line and the black line represent the utility of type 1 academics and type 2 academics as a function of  $\pi_i$  respectively. The solid red line represents  $\pi_A$  and its intersections with the blue and black lines show the utilities of the academics in department A. Similarly, the dashed red line represents  $\pi_B$  and its intersections with the blue and black lines show the utilities of the academics in department B. The key feature of the picture is that the blue line is non-decreasing in  $\pi_i$  and the black line is non-increasing in  $\pi_i$ . Therefore, academics of type 1 in department A can increase their utility by moving to department B and academics of type 2 in department B can increase their utility by moving to department A. At the end of the swapping process, when there are no pairs of academics willing to move, we have that in department A there are 2 academics of type 1 and 23 academics of type 2 while in department B we have 25 academics of type 1 and 0 academics of type 2. This implies  $\pi_A = \frac{2}{25}$  and  $\pi_B = 1$ . Given the probability thresholds, by Theorem 2 we have that the optimal targets correspond to the case of full specialization in teaching in department A and full specialization in research in department B.

The next example shows that the previous result depends on  $\rho$  and that when it is low enough academics may not find optimal to move between departments.

**Example 3.** Consider the types of academics given in Example 1 and let  $\rho = \frac{3}{10}$ . Consider the departments A and B described in Example 2. As above, the optimal targets consist in diversification skewed toward teaching in department A and diversification skewed toward research in department B. To study if academics swap departments we consider the functions describing the utility of the two types of academics for different  $\pi_i$  in Figure 3b. We then observe that both types of academics in department A are willing to move to B but no academics in B is willing to move to A. Therefore, there will be no move of academics between the departments and their optimal targets will remain diversification.

The two examples show that the use of targets does not necessarily lead department to specialize in just research or teaching. The analysis conducted by considering Figures 3a and 3b shows the important role of the initial shares  $\pi_A$ ,  $\pi_B$ , and of the parameter  $\rho$  in determining academics mobility. Heuristically, when  $\rho$  is close to 1, i.e., high engagement with the targets, academics are willing to go where targets are skewed toward their most productive activity. While, for  $\rho$  closer to  $\beta$ , the use of targets is generally utility detrimental and, consequently, there are less incentives to change department.



(a) Academics' utilities with  $\rho = \frac{9}{10}$ .

(b) Academics' utilities with  $\rho = \frac{3}{10}$ .

Figure 3: Utilities for the two types of academics for different values of  $\pi_i$ .

#### 4.4 Endogenous (total) effort supply

In our last example, we consider the case in which academics are not coerced to work and they can decide the amount of total effort to supply to produce research and teaching. In other words, the question is: “what are the effects of the participation constraints on the optimal targets?”.

To study this problem, we modify the academics' utility function with the MS normalization, defined in (9), by introducing the cost of effort in a linear way. Let  $l_i$  be the total supply of effort and  $\chi$  be the unit cost of effort. The academics' effort on research is as usual  $e_i$  while the effort on teaching is now  $l_i - e_i$ . The maximization problem of an academic of type  $i$  becomes then

$$\begin{aligned} \max_{e_i, E_i} \quad & \left( \frac{1}{2}\theta^\rho + \frac{1}{2} \right)^{\frac{1}{\rho} - \frac{1}{\beta}} \left( \frac{1}{2}\theta^{\rho-\beta} (m_i e_i)^\beta + \frac{1}{2} (n_i (l_i - e_i))^\beta \right)^{\frac{1}{\beta}} - \chi l_i, \\ \text{s.t.} \quad & e_i \in [0, l_i], \\ & l_i \in [0, E]. \end{aligned} \quad (13)$$

The solution to this maximization problem with costly supply of effort is given by the pair

$$(e_i^*, l_i^*) = \begin{cases} \left( \frac{E}{1+a_i\theta^b}, E \right) & \text{if } R_i(\theta) > \chi \\ \left( \frac{l_i}{1+a_i\theta^b}, l_i \right) \text{ with } l_i \in [0, E] & \text{if } R_i(\theta) = \chi \\ (0, 0) & \text{if } R_i(\theta) < \chi \end{cases}, \quad (14)$$

with  $R_i(\theta) = \left( \frac{1}{2}\theta^\rho + \frac{1}{2} \right)^{\frac{1}{\rho}} \left[ \frac{1}{2}\theta^{\rho-\beta} \left( \frac{m_i}{1+a_i\theta^b} \right)^\beta + \frac{1}{2} \left( n_i \left( 1 - \frac{1}{1+a_i\theta^b} \right) \right)^\beta \right]^{\frac{1-\beta}{\beta}} \frac{1}{2} n_i^\beta \left( 1 - \frac{1}{1+a_i\theta^b} \right)^{\beta-1}$ .<sup>16</sup>

The optimal supply of effort is then the full amount  $E$  or nothing. Therefore, the participation constraints require that the targets chosen by the manager are such that both agents have a utility greater than 0, the academics' utility reservation level. By writing the

<sup>16</sup>Proposition 3 in the Appendix shows the detailed calculation on how to derive the maximum and  $R_i(\theta)$ .

participation constraints in terms of  $R_i(\theta)$ , the manager's maximization problem becomes

$$\begin{aligned} \sup_{\theta} \quad & N(\pi(p_x x_1(\theta)) + p_y y_1(\theta)) + (1 - \pi)(p_x x_2(\theta) + p_y y_2(\theta)), \\ \text{s.t.} \quad & R_1(\theta) \geq \chi, \\ & R_2(\theta) \geq \chi. \end{aligned} \tag{15}$$

We denote the optimal targets that solves the maximization problem with the participation constraints by  $\theta_c^*$ .

Unfortunately, it is not possible to solve this problem analytically and we then consider an example to see how the optimal targets are affected by the participation constraints.

**Example 4.** Consider the same types of academics, department and external incentives in Example 1. Academics have the utility functions with the MS normalization defined in the maximization problem (13) and let  $\chi = 0.2$  and  $\rho = 0.6$ .

Our analysis is based on Figures 4a and 4b. In Figure 4a, the black line and blue line represent the functions  $R_i(\theta) - \chi$  for academics of type 1 and type 2 respectively. The points in which the two functions cross the horizontal axis gives us  $[\theta_{min}, \theta_{max}]$  which is the interval of optimal targets satisfying the participation constraints. It is immediate to note that the unconstrained optimal target  $\theta^*$  does not respect the participation constraint of type 1 academics since it is lower than  $\theta_{min}$ . Figure 4b depicts the manager's payoff as a function of  $\theta$  and it shows that the constrained optimal targets is  $\theta_c^* = \theta_{min}$ . In other words, the manager maximizes the total output under the participation constraints by moving the optimal target from  $\theta^*$  to  $\theta_{min}$ .

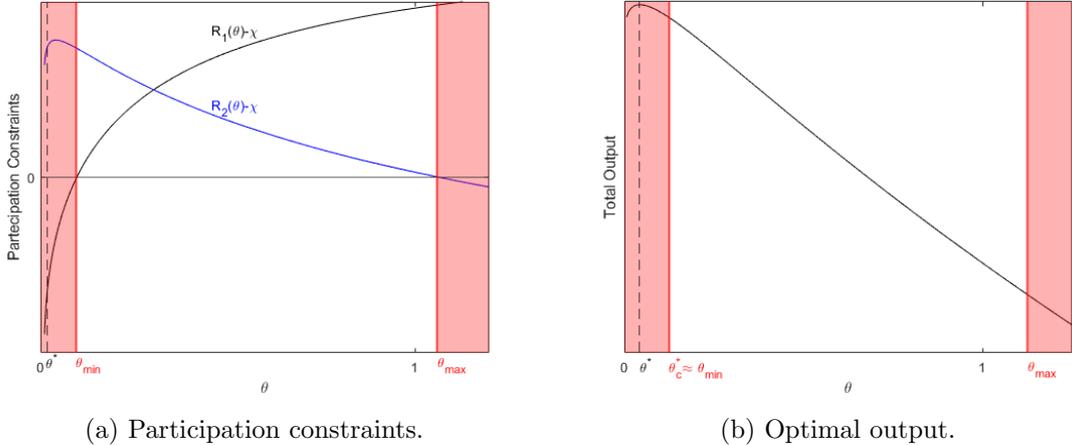


Figure 4: Participation constraints and constrained optimal targets.

As in the previous example, we further investigate the role of  $\rho$ . Table 4 shows  $\theta^*$  and  $\theta_c^*$  for different values of  $\rho$ .

Interestingly, we can see that the optimal targets and the constrained optimal targets are becoming closer when  $\rho$  increases. We can interpret this as the fact that for higher values of  $\rho$  academics are more engaged with the targets and then the participation constraints become more loosely.

This example highlights again the key role of  $\rho$  in our analysis and how the participation constraints change the manager's optimal targets.

	$\approx \beta$	$2\beta$	$\rho =$		
			$3\beta$	$4\beta$	$\approx 5\beta$
$\theta^*$	$2.8e^{-35}$	$2.8e^{-5}$	0.0069	0.0353	0.08
$\theta_c^*$	$1.5e^{-13}$	0.038	0.085	0.105	0.073

Table 4: Constrained and Unconstrained Optimal Targets.

## 5 Conclusion

This paper has presented a model of optimal research and teaching targets for the academic sector. The theory brings together ideas from the motivated agents and reference points literature and adapt them to describe an academic environment characterized by two types of academics and a manager. The analysis leads to a rich set of predictions. After identifying the main determinants in deciding optimally the targets, the analysis clarifies what are the implications for the manager and the academics to work in different settings by answering the following questions: Is academic freedom always better? What about switching from common to personalized targets? Can diversification between research and teaching be optimal also with labor mobility between universities? Could an academic stop working in response to targets? The answers to these questions have been rigorously derived and may help to design policies in the academic sector.

There are numerous ways the analysis can be extended. One of the most interesting would be to use the theoretical results on optimal research and teaching targets found in this paper and to assess how far from the optimum is the actual targeting of the universities. Such a comparison would inform an university on how to tune its existing policy and improve its research and teaching output. The main challenge of this extension lies on the universities' reluctance of sharing information about research and teaching targets which is often considered an internal policy not meant to become of public domain.

## A Appendix: Proofs

### A.1 Academics' optimal effort

We now prove a proposition on the function  $e_i^*(\theta)$  which characterizes the academics' optimal effort.

**Proposition 1.** For each  $\theta \in [0, \infty)$ , the solution of the maximization problem (4) is given by  $e_i^*(\theta) = \frac{E}{1+a_i\theta^b}$ , with  $a_i = \left(\frac{m_i}{n_i}\right)^{\frac{\beta}{\beta-1}} > 0$  and  $b = \frac{\rho-\beta}{\beta-1} < -1$ . Furthermore,  $e_i^*(0) = 0$ ,  $\lim_{\theta \rightarrow \infty} e_i^*(\theta) = E$ ,  $\frac{de_i^*(\theta)}{d\theta} > 0$  for each  $\theta \in (0, \infty)$ , and  $\frac{de_i^*(0)}{d\theta} = 0$ .

*Proof.* Let  $u_i(e_i) = u_i(x_i(e_i), y_i(e_i); r_x, r_y)$ . After some simplifications, the first order necessary conditions for a maximum of (4) can be written as

$$\frac{du_i(e_i)}{de_i} = r_x^{\rho-\beta} (m_i e_i)^{\beta-1} m_i - r_y^{\rho-\beta} (n_i (E - e_i))^{\beta-1} n_i = 0.$$

By rearranging the terms, this also implies that the marginal rate of substitution between research and teaching is equal to the ratio of productivities. By solving the first order condition, we find the  $e_i^*$  defined in (5) is a critical point. Next, observe that  $e_i^*(\theta)$  corresponds

to a local maximum since some tedious computations confirm that

$$\frac{d^2 u_i(e_i^*)}{d^2 e_i} = \frac{1}{2} A(r_x, r_y) (\beta - 1) u_i(e_i^*)^{\frac{1-\beta}{\beta}} \left( r_x^{\rho-\beta} m^\beta e_i^{*\beta-2} + r_y^{\rho-\beta} n^\beta (E - e_i^*)^{\beta-2} \right) < 0.$$

This follows by the fact that  $-\infty < \beta < 1$ ,  $A(r_x, r_y) > 0$ , and  $r_x, r_y \geq 0$ . Then, by letting  $\theta = \frac{r_x}{r_y}$ , we can easily define the function  $e_i^*(\theta)$  in (6). Furthermore, it is immediate to see that  $e_i^*(0) = 0$  and  $\lim_{\theta \rightarrow \infty} e_i^*(\theta) = E$  since (6) can be written as  $e_i^*(\theta) = \frac{\theta^{|\beta|} E}{\theta^{|\beta|} + a_i}$ . Finally,  $\frac{de_i^*(\theta)}{d\theta} = -\frac{ba_i \theta^{b-1}}{(1+a_i \theta^b)^2} E = \frac{|b| a_i \theta^{-b-1}}{(\theta^{|\beta|} + a_i)^2} E$ . But then, it follows that  $\frac{de_i^*(\theta)}{d\theta} > 0$  for each  $\theta \in (0, \infty)$  as  $b < -1$  and that  $\frac{de_i^*(0)}{d\theta} = 0$  as  $-b - 1 > 0$ .  $\square$

The next proposition clarifies why at the optimal targets  $\theta^*$  the parameter  $\rho$  does not influence the optimal allocation of effort between research and teaching.

**Proposition 2.** Let  $\frac{p_x}{p_y} \in (\frac{n_1}{m_1}, \frac{n_2}{m_2})$ ,  $\epsilon \in (1, \infty)$  and  $\pi \in (\tilde{\pi}_1, \tilde{\pi}_3)$ . Given the optimal targets  $\theta^* \in (0, \infty)$  and any pair  $(r_x^*, r_y^*)$  such that  $\theta^* = \frac{r_x^*}{r_y^*}$ , then the function representing the indifference curve passing through the point  $(x_i(\theta^*), y_i(\theta^*))$  does not depend on  $\rho$ .

*Proof.* Given the  $\theta^*$  and the corresponding pairs  $(r_x^*, r_y^*)$  and  $(x_i(\theta^*), y_i(\theta^*))$ , the utility of an academic of type  $i$  is given by

$$A(r_x^*, r_y^*) r_y^{*\rho-\beta} \left( \theta^{*\rho-\beta} x_i(\theta^*)^\beta + y_i(\theta^*)^\beta \right)^{\frac{1}{\beta}} = U^*$$

Then, the indifference curve is given by all the pairs  $(x_i, y_i)$  such that

$$\underbrace{A(r_x^*, r_y^*) r_y^{*\rho-\beta} (\theta^{*\rho-\beta} x_i^\beta + y_i^\beta)^{\frac{1}{\beta}}}_{\equiv u_i(x_i, y_i; r_x^*, r_y^*)} = \underbrace{A(r_x^*, r_y^*) r_y^{*\rho-\beta} (\theta^{*\rho-\beta} x_i(\theta^*)^\beta + y_i(\theta^*)^\beta)^{\frac{1}{\beta}}}_{\equiv U^*}$$

or equivalently

$$(\theta^{*\rho-\beta} x_i^\beta + y_i^\beta)^{\frac{1}{\beta}} = (\theta^{*\rho-\beta} x_i(\theta^*)^\beta + y_i(\theta^*)^\beta)^{\frac{1}{\beta}}$$

Given the result in (8), let's define  $\theta^* = K^{\frac{\beta-1}{\rho-\beta}}$  with  $K = \frac{-a_2 \pi k_1 - a_1 (1-\pi) k_2 + (a_1 - a_2) \sqrt{-\pi k_1 (1-\pi) k_2}}{a_2^2 \pi k_1 + a_1^2 (1-\pi) k_2}$  not depending on  $\rho$ . By substituting this in the equation above we obtain

$$(K^{\beta-1} x_i^\beta + y_i^\beta)^{\frac{1}{\beta}} = (K^{\beta-1} x_i(\theta^*)^\beta + y_i(\theta^*)^\beta)^{\frac{1}{\beta}}$$

As we remarked at the end of Subsection 3.2, it is immediate to verify that  $x_i(\theta^*)$  and  $y_i(\theta^*)$  do not depend on  $\rho$  and, therefore, the indifference curve passing through the point  $(x(\theta^*), y(\theta^*))$  does not depend on  $\rho$ .  $\square$

The next proposition shows the academics' optimal effort in the case of Subsection 4.4 where we have introduced the participation constraints.

**Proposition 3.** The solution of the maximization problem (13) is given by the result in (14).

*Proof.* To solve the academic maximization problem we set the following Lagrangian, which includes the multipliers for all constraints,

$$\begin{aligned} \mathcal{L}(e_i, l_i) = & \left( \frac{1}{2} \theta^\rho + \frac{1}{2} \right)^{\frac{1}{\rho} - \frac{1}{\beta}} \left( \frac{1}{2} \theta^{\rho-\beta} (m_i e_i)^\beta + \frac{1}{2} (n_i (l_i - e_i))^\beta \right)^{\frac{1}{\beta}} - \chi l_i \\ & - \lambda_1 (e_i - l_i) - \lambda_2 (l_i - E) + \mu_1 e_i + \mu_2 l_i. \end{aligned}$$

By applying the Kuhn-Tucker Theorem, we have the following necessary conditions for a maximum:

$$\frac{\partial \mathcal{L}}{\partial e_i} = \left(\frac{1}{2}\theta^\rho + \frac{1}{2}\right)^{\frac{1}{\rho}} \left(\frac{1}{2}\theta^{\rho-\beta}(m_i e_i)^\beta + \frac{1}{2}(n_i(l_i - e_i))^\beta\right)^{\frac{1-\beta}{\beta}} \left(\frac{1}{2}\theta^{\rho-\beta}m_i^\beta e_i^{\beta-1} - \frac{1}{2}n_i^\beta(l_i - e_i)^{\beta-1}\right) - \lambda_1 + \mu_1 = 0$$

$$\frac{\partial \mathcal{L}}{\partial l_i} = \left(\frac{1}{2}\theta^\rho + \frac{1}{2}\right)^{\frac{1}{\rho}} \left[\frac{1}{2}\theta^{\rho-\beta}(m_i e_i)^\beta + \frac{1}{2}(n_i(l_i - e_i))^\beta\right]^{\frac{1-\beta}{\beta}} \frac{1}{2}n_i^\beta(l_i - e_i)^{\beta-1} - \chi + \lambda_1 - \lambda_2 + \mu_2 = 0$$

$$\mu_1 e_i = 0, \mu_2 l_i = 0, \lambda_1(e_i - l_i) = 0 \text{ and } \lambda_2(l_i - E) = 0$$

$$-\mu_1 \geq 0, -\mu_2 \geq 0, \lambda_1 \geq 0 \text{ and } \lambda_2 \geq 0$$

$$e_i \in [0, l_i] \text{ and } l_i \in [0, E]$$

. Given the aim of our analysis, we just consider the solutions for which  $l_i^* > 0$ . Hence, we have  $\mu_2^* = 0$ . Moreover, as the non-linear part of the academics' utility function is a CES function defined over research and teaching we must also have that  $\mu_1^* = 0$  and  $\lambda_1^* = 0$ . Thus, by solving the first equation, we obtain  $e_i^* = \frac{l_i}{1+a_i\theta^b}$  and, by substituting it in the second equation, we get

$$\underbrace{\left(\frac{1}{2}\theta^\rho + \frac{1}{2}\right)^{\frac{1}{\rho}} \left[\frac{1}{2}\theta^{\rho-\beta} \left(\frac{m_i}{1+a_i\theta^b}\right)^\beta + \frac{1}{2} \left(n_i \left(1 - \frac{1}{1+a_i\theta^b}\right)\right)^\beta\right]^{\frac{1-\beta}{\beta}} \frac{1}{2}n_i^\beta \left(1 - \frac{1}{1+a_i\theta^b}\right)^{\beta-1}}_{\equiv R_i(\theta)} - \chi = \lambda_2^*.$$

Therefore, for  $R_i(\theta) > \chi$  we have that  $\lambda_2^* > 0$  which implies  $l_i^* = E$ . Next, it is straightforward to verify that for  $R_i(\theta) = \chi$  we have that  $l_i^* \in [0, E]$  and for  $R_i(\theta) < \chi$  we have that  $l_i^* = 0$ . As in Proposition 1, it is possible to check that the second order conditions for a maximum are satisfied.  $\square$

## A.2 Proof of Theorems 1 and 2

We now report the proof of Theorem 1 for the case of skewed external incentives.

*Proof of Theorem 1.* When  $\frac{p_x}{p_y} \in (-\infty, \frac{n_1}{m_1}]$ , we have that  $\frac{n_2}{m_2} > \frac{n_1}{m_1} \geq \frac{p_x}{p_y}$  which implies  $\tilde{n}_1 \geq \tilde{m}_1$  and  $\tilde{n}_2 > \tilde{m}_2$  with  $\tilde{m}_i = p_x m_i$  and  $\tilde{n}_i = p_y n_i$ , for  $i = 1, 2$ . Then,

$$\frac{dV(\theta)}{d\theta} = \pi(\tilde{m}_1 - \tilde{n}_1) \frac{de_1(\theta)}{d\theta} + (1 - \pi)(\tilde{m}_2 - \tilde{n}_2) \frac{de_2(\theta)}{d\theta} < 0,$$

for each  $\theta \in (0, \infty)$  and  $\frac{dV(0)}{d\theta} = 0$  by the results on  $\frac{de_i(\theta)}{d\theta}$  in Proposition 1. Therefore, the manager payoff function is strictly decreasing in  $\theta$  and his optimal strategy is  $\theta^* = 0$ .

When  $\frac{p_x}{p_y} \in [\frac{n_2}{m_2}, \infty)$ , we have that  $\frac{p_x}{p_y} \geq \frac{n_2}{m_2} > \frac{n_1}{m_1}$  which implies  $\tilde{m}_1 > \tilde{n}_1$  and  $\tilde{m}_2 \geq \tilde{n}_2$ . Following the same steps above, this implies  $\frac{dV(\theta)}{d\theta} > 0$  for each  $\theta \in (0, \infty)$  and  $\frac{dV(0)}{d\theta} = 0$ . Therefore, the manager payoff function is strictly increasing in  $\theta$  and his optimal strategy is  $\theta^* \rightarrow \infty$ .  $\square$

The following lemmas are required to prove Theorem 2 for the case of balanced external incentives.

**Lemma 1.** Let  $\frac{p_x}{p_y} \in (\frac{n_1}{m_1}, \frac{n_2}{m_2})$  and consider the following constants:

$$\tilde{\pi}_1 = \frac{(\tilde{n}_2 - \tilde{m}_2)a_1}{(\tilde{n}_2 - \tilde{m}_2)a_1 + (\tilde{m}_1 - \tilde{n}_1)a_2}, \quad \tilde{\pi}_2 = \frac{\tilde{n}_2 - \tilde{m}_2}{\tilde{n}_2 - \tilde{m}_2 + \tilde{m}_1 - \tilde{n}_1}, \quad \tilde{\pi}_3 = \frac{(\tilde{n}_2 - \tilde{m}_2)a_2}{(\tilde{n}_2 - \tilde{m}_2)a_2 + (\tilde{m}_1 - \tilde{n}_1)a_1},$$

where  $\tilde{m}_i = p_x m_i$  and  $\tilde{n}_i = p_y n_i$ , for  $i = 1, 2$ . Then, the following relations hold:

- a)  $0 < \tilde{\pi}_3 < \tilde{\pi}_2 < \tilde{\pi}_1 < 1$  when  $\beta \in (-\infty, 0)$ .
- b)  $0 < \tilde{\pi}_3 = \tilde{\pi}_2 = \tilde{\pi}_1 < 1$  when  $\beta = 0$ .
- c)  $0 < \tilde{\pi}_1 < \tilde{\pi}_2 < \tilde{\pi}_3 < 1$  when  $\beta \in (0, 1)$ .

*Proof.* We start by noting that  $\frac{p_x}{p_y} \in (\frac{n_1}{m_1}, \frac{n_2}{m_2})$  implies that  $\frac{p_x}{p_y} > \frac{n_1}{m_1}$  and  $\frac{p_x}{p_y} < \frac{n_2}{m_2}$ . Then,  $\tilde{m}_1 > \tilde{n}_1$  and  $\tilde{n}_2 > \tilde{m}_2$ .

- a) If  $\beta \in (-\infty, 0)$ , it follows that  $\frac{\beta}{\beta-1} > 0$ . Then  $(\frac{\tilde{m}_1}{\tilde{n}_1})^{\frac{\beta}{\beta-1}} > (\frac{\tilde{m}_2}{\tilde{n}_2})^{\frac{\beta}{\beta-1}}$ . But then,  $a_1 > a_2$ . Hence,  $0 < \tilde{\pi}_3 < \tilde{\pi}_2 < \tilde{\pi}_1 < 1$ .
- b) If  $\beta = 0$ , it follows that  $\frac{\beta}{\beta-1} = 0$ . Then  $(\frac{\tilde{m}_1}{\tilde{n}_1})^{\frac{\beta}{\beta-1}} = (\frac{\tilde{m}_2}{\tilde{n}_2})^{\frac{\beta}{\beta-1}}$ . But then,  $a_1 = a_2$ . Hence,  $0 < \tilde{\pi}_3 = \tilde{\pi}_2 = \tilde{\pi}_1 < 1$ .
- c) If  $\beta \in (0, 1)$ , it follows that  $\frac{\beta}{\beta-1} < 0$ . By following the same steps above, we obtain that  $a_2 > a_1$  and we then conclude that  $0 < \tilde{\pi}_1 < \tilde{\pi}_2 < \tilde{\pi}_3 < 1$ .

□

Our study of the manager's optimal targets is based on the function  $\tilde{V}(\theta)$  defined in the next lemma.

**Lemma 2.** Let

$$\tilde{V}(\theta) = \pi(\tilde{m}_1 - \tilde{n}_1) \frac{E}{1 + a_1 \theta^b} + (1 - \pi)(\tilde{m}_2 - \tilde{n}_2) \frac{E}{1 + a_2 \theta^b}.$$

The critical points of  $\tilde{V}(\theta)$  are the same of  $V(\theta)$ .

*Proof.* It is straightforward to see that  $\tilde{V}(\theta) = V(\theta) - (\pi \tilde{n}_1 + (1 - \pi) \tilde{n}_2)E$ . Then,  $\tilde{V}(\theta)$  is a linear transformation of  $V(\theta)$  which implies that the critical points of the two functions are the same. □

The next lemma studies the positive critical points of  $\tilde{V}(\theta)$

**Lemma 3.** Let  $\frac{p_x}{p_y} \in (\frac{n_1}{m_1}, \frac{n_2}{m_2})$ . Then, we have that

- a) Let  $\beta \in (-\infty, 0)$ .
  - If  $\pi \in (0, \tilde{\pi}_3]$  or  $\pi \in [\tilde{\pi}_1, 1)$ , then there are no positive critical points for  $\tilde{V}(\theta)$ .
  - If  $\pi \in (\tilde{\pi}_3, \tilde{\pi}_1)$  then it exists a unique positive critical point for  $\tilde{V}(\theta)$ , which is the local and global minimum.
- b) Let  $\beta = 0$ . For any  $\pi \in (0, 1)$  there are no positive critical points for  $\tilde{V}(\theta)$ .
- c) Let  $\beta \in (0, 1)$ 
  - If  $\pi \in (0, \tilde{\pi}_1]$  or  $\pi \in [\tilde{\pi}_3, 1)$ , then there are no positive critical points for  $\tilde{V}(\theta)$ .
  - If  $\pi \in (\tilde{\pi}_1, \tilde{\pi}_3)$  then it exists a unique critical point for  $\tilde{V}(\theta)$ , which is the local and global maximum.

*Proof.* Since  $\frac{p_x}{p_y} \in (\frac{n_1}{m_1}, \frac{n_2}{m_2})$ , we have that  $\tilde{m}_1 > \tilde{n}_1$  and  $\tilde{n}_2 > \tilde{m}_2$ , as shown in Lemma 1. Next, we consider the first order condition of  $\tilde{V}(\theta)$

$$\frac{d\tilde{V}(\theta)}{d\theta} = -b\theta^{b-1}E \left[ \pi(m_1 - n_1) \frac{a_1}{(1 + a_1\theta^b)^2} + (1 - \pi)(m_2 - n_2) \frac{a_2}{(1 + a_2\theta^b)^2} \right] = 0,$$

which can be written as

$$\frac{d\tilde{V}(\theta)}{d\theta} = E|b|\theta^{b-1} \left[ \frac{A\theta^{2b} + B\theta^b + C}{(1 + a_1\theta^b)^2(1 + a_2\theta^b)^2} \right] = 0, \quad (16)$$

with

$$\begin{aligned} A &= a_1a_2 [\pi(\tilde{m}_1 - \tilde{n}_1)a_2 + (1 - \pi)(\tilde{m}_2 - \tilde{n}_2)a_1], \\ B &= 2a_1a_2 [\pi(\tilde{m}_1 - \tilde{n}_1) + (1 - \pi)(\tilde{m}_2 - \tilde{n}_2)], \\ C &= \pi(\tilde{m}_1 - \tilde{n}_1)a_1 + (1 - \pi)(\tilde{m}_2 - \tilde{n}_2)a_2. \end{aligned}$$

Therefore, the critical points of  $\tilde{V}(\theta)$  are the roots of the parabola  $A\theta^{2b} + B\theta^b + C$  in the variable  $\theta^b$ . Since the discriminant is always non-negative, i.e.,

$$\Delta = B^2 - 4AC = 4a_1a_2\pi(1 - \pi)(\tilde{n}_1 - \tilde{m}_1)(\tilde{m}_2 - \tilde{n}_2)(a_1 - a_2)^2 \geq 0$$

then the roots of the parabola are always real. Before proceeding, it is immediate to verify that

$$\begin{aligned} A > 0 &\Leftrightarrow \pi > \tilde{\pi}_1 \quad \text{and} \quad A = 0 \Leftrightarrow \pi = \tilde{\pi}_1, \\ B > 0 &\Leftrightarrow \pi > \tilde{\pi}_2 \quad \text{and} \quad B = 0 \Leftrightarrow \pi = \tilde{\pi}_2, \\ C > 0 &\Leftrightarrow \pi > \tilde{\pi}_3 \quad \text{and} \quad C = 0 \Leftrightarrow \pi = \tilde{\pi}_3. \end{aligned}$$

Furthermore, the two roots are

$$\theta_1^{*b} = \frac{-B - \sqrt{\Delta}}{2A} \quad \text{and} \quad \theta_2^{*b} = \frac{-B + \sqrt{\Delta}}{2A}.$$

a) Let  $\beta \in (-\infty, 0)$ . Then,  $0 < \tilde{\pi}_3 < \tilde{\pi}_2 < \tilde{\pi}_1 < 1$  by Lemma 1. We need to consider the following cases.

If  $\pi \in (0, \tilde{\pi}_3)$ , then  $A < 0$ ,  $B < 0$ , and  $C < 0$ . Since  $A < 0$  and  $C < 0$ , it follows that  $B^2 > \Delta$ . Hence,  $\theta_1^{*b} < 0$  and  $\theta_2^{*b} < 0$  and there are no positive critical points for  $\tilde{V}(\theta)$ .

If  $\pi = \tilde{\pi}_3$ , then  $A < 0$ ,  $B < 0$ , and  $C = 0$ . Since  $A < 0$  and  $C = 0$ , it follows that  $B^2 = \Delta$ . Hence,  $\theta_1^{*b} = 0$  and  $\theta_2^{*b} < 0$ . However, given the first order condition in (16),  $\theta_1^{*b} = 0$  cannot be a critical point of  $\tilde{V}(\theta)$ .

If  $\pi \in (\tilde{\pi}_3, \tilde{\pi}_2]$ , then  $A < 0$ ,  $B \leq 0$ , and  $C > 0$ . Since  $A < 0$  and  $C > 0$ , it follows that  $B^2 < \Delta$ . Hence,  $\theta_1^{*b} > 0$  and  $\theta_2^{*b} < 0$ . Given the hump-shaped parabola for  $A < 0$ , we have that  $\frac{d\tilde{V}(\theta)}{d\theta} > 0$  if  $\theta^b \in (0, \theta_1^{*b})$  and  $\frac{d\tilde{V}(\theta)}{d\theta} < 0$  if  $\theta^b \in (\theta_1^{*b}, \infty)$ . Since  $b < -1$ , when we consider the original variable  $\theta$  we obtain that  $\frac{d\tilde{V}(\theta)}{d\theta} < 0$  if  $\theta \in (0, \theta_1^*)$  and  $\frac{d\tilde{V}(\theta)}{d\theta} > 0$  if  $\theta \in (\theta_1^*, \infty)$ . Therefore,  $\theta_1^*$  is a local minimum and a global minimum of  $\tilde{V}(\theta)$  in the positive domain.

If  $\pi \in (\tilde{\pi}_2, \tilde{\pi}_1)$ , then  $A < 0$ ,  $B > 0$ , and  $C > 0$ . Since  $A < 0$  and  $C > 0$ , it follows that

$B^2 < \Delta$ . Hence,  $\theta_1^{*b} > 0$  and  $\theta_2^{*b} < 0$ . Given the hump-shaped parabola for  $A < 0$ , we can follow the same steps above and we conclude that  $\theta_1^*$  is a local minimum and a global minimum of  $\tilde{V}(\theta)$  in the positive domain.

If  $\pi = \tilde{\pi}_1$ , then  $A = 0$ ,  $B > 0$ , and  $C > 0$ . Since  $A = 0$ , the parabola becomes a line represented by the equation  $B\theta^b + C$ . Hence,  $\theta^* = -\frac{C}{B} < 0$  and there are no positive critical points for  $\tilde{V}(\theta)$ .

If  $\pi \in (\tilde{\pi}_1, 1)$ , then  $A > 0$ ,  $B > 0$ , and  $C > 0$ . Since  $A > 0$  and  $C > 0$ , it follows that  $B^2 > \Delta$ . Hence,  $\theta_1^{*b} < 0$  and  $\theta_2^{*b} < 0$  and there are no positive critical points for  $\tilde{V}(\theta)$ .

b) Let  $\beta = 0$ .<sup>17</sup> Then,  $0 < \tilde{\pi}_3 = \tilde{\pi}_2 = \tilde{\pi}_1 < 1$  by Lemma 1. We need to consider the following cases.

If  $\pi \in (0, \tilde{\pi}_3)$ , then  $A < 0$ ,  $B < 0$ , and  $C < 0$ . As above, there are no positive critical points for  $\tilde{V}(\theta)$ .

If  $\pi = \tilde{\pi}_3 = \tilde{\pi}_2 = \tilde{\pi}_1$ , then  $A = 0$ ,  $B = 0$ , and  $C = 0$ . By observing the first order condition in (16), we can conclude that in such a case the function  $\tilde{V}(\theta)$  becomes a constant function. We would say that there are no critical points.

If  $\pi \in (\tilde{\pi}_1, 1)$ , then  $A > 0$ ,  $B > 0$ , and  $C > 0$ . As above, there are no positive critical points for  $\tilde{V}(\theta)$ .

c) Let  $\beta \in (0, 1)$ . Then,  $0 < \tilde{\pi}_1 < \tilde{\pi}_2 < \tilde{\pi}_3 < 1$  by Lemma 1. We need to consider the following cases.

If  $\pi \in (0, \tilde{\pi}_1)$ , then  $A < 0$ ,  $B < 0$ , and  $C < 0$ . As above, we can conclude that there are no positive critical points for  $\tilde{V}(\theta)$ .

If  $\pi = \tilde{\pi}_1$ , then  $A = 0$ ,  $B < 0$ , and  $C < 0$ . Since  $A = 0$ , the parabola becomes a line and, as above, we conclude that there are no positive critical points for  $\tilde{V}(\theta)$ .

If  $\pi \in (\tilde{\pi}_1, \tilde{\pi}_2]$ , then  $A > 0$ ,  $B \leq 0$ , and  $C < 0$ . Since  $A > 0$  and  $C < 0$ , it follows that  $B^2 < \Delta$ . Hence,  $\theta_1^{*b} < 0$  and  $\theta_2^{*b} > 0$ . Given the U-shaped parabola for  $A > 0$ , we have that  $\frac{d\tilde{V}(\theta)}{d\theta} < 0$  if  $\theta^b \in (0, \theta_2^{*b})$  and  $\frac{d\tilde{V}(\theta)}{d\theta} > 0$  if  $\theta^b \in (\theta_2^{*b}, \infty)$ . Since  $b < -1$ , when we consider the original variable  $\theta$  we obtain that  $\frac{d\tilde{V}(\theta)}{d\theta} > 0$  if  $\theta \in (0, \theta_2^*)$  and  $\frac{d\tilde{V}(\theta)}{d\theta} < 0$  if  $\theta \in (\theta_2^*, \infty)$ . Therefore,  $\theta_2^*$  is a local maximum and a global maximum of  $\tilde{V}(\theta)$  in the positive domain.

If  $\pi \in (\tilde{\pi}_2, \tilde{\pi}_3)$ , then  $A > 0$ ,  $B > 0$ , and  $C < 0$ . Since  $A > 0$  and  $C < 0$ , it follows that  $B^2 < \Delta$ . Hence,  $\theta_1^{*b} < 0$  and  $\theta_2^{*b} > 0$ . Given the U-shaped parabola for  $A > 0$ , we can follow the same steps above and conclude that  $\theta_2^*$  is a local maximum and a global maximum of  $\tilde{V}(\theta)$  in the positive domain.

If  $\pi = \tilde{\pi}_3$ , then  $A > 0$ ,  $B > 0$ , and  $C = 0$ . Since  $A > 0$  and  $C = 0$ , it follows that  $B^2 = \Delta$ . Hence,  $\theta_1^{*b} < 0$  and  $\theta_2^{*b} = 0$ . However, given the first order condition in (16),  $\theta_2^{*b} = 0$  cannot be a critical point of  $\tilde{V}(\theta)$ .

If  $\pi \in (\tilde{\pi}_3, 1)$ , then  $A > 0$ ,  $B > 0$ , and  $C > 0$ . As above, we can conclude that there are no positive critical points for  $\tilde{V}(\theta)$ .

By the cases (a), (b), and (c), the results of the lemma follow easily.  $\square$

<sup>17</sup>Note that in this case the academics' utility function is well defined only for some normalization  $A(r_x, r_y)$  such as the MS normalization.

**Lemma 4.** The following relationships on the limits of  $V(\theta)$  hold

- $V(0) > \lim_{\theta \rightarrow \infty} V(\theta)$  when  $\pi < \tilde{\pi}_2$ ,
- $V(0) = \lim_{\theta \rightarrow \infty} V(\theta)$  when  $\pi = \tilde{\pi}_2$ ,
- $V(0) < \lim_{\theta \rightarrow \infty} V(\theta)$  when  $\pi > \tilde{\pi}_2$ .

*Proof.* First, note that  $V(0) = (\pi n_1 + (1 - \pi)n_2)p_y E$  and that  $\lim_{\theta \rightarrow \infty} V(\theta) = (\pi m_1 + (1 - \pi)m_2)p_x E > 0$ . Then, the relationships in the lemma follow straightforwardly by the definition of  $\tilde{\pi}_2$ .  $\square$

We can now prove Theorem 2.

*Proof of Theorem 2.* Let  $\frac{p_x}{p_y} \in (\frac{n_1}{m_1}, \frac{n_2}{m_2})$ . The proof is based on the cases considered in Lemma 3. Remember that we denote by  $\theta^*$  the superior of the manager's payoff function.

- Let  $\epsilon \in (0, 1]$  which implies  $\beta \in (-\infty, 0]$ . We have three cases.

If  $\pi \in (0, \tilde{\pi}_2)$ , by Lemmas 2 and 3, it follows that  $V(\theta)$  has no global maximum on the non-negative domain. By Lemma 4, we have that  $0 < \lim_{\theta \rightarrow \infty} V(\theta) < V(0) < \infty$ . Hence,  $\theta^* = 0$ .

If  $\pi = \tilde{\pi}_2$ , by Lemmas 2 and 3, it follows that  $V(\theta)$  has no global maximum on the non-negative domain. By Lemma 4, we have that  $0 < \lim_{\theta \rightarrow \infty} V(\theta) = V(0) < \infty$ . Hence,  $\theta^* = 0$  and  $\theta^* \rightarrow \infty$ .

If  $\pi \in (\tilde{\pi}_2, 1)$ , by Lemmas 2 and 3, it follows that  $V(\theta)$  has no global maximum on the non-negative domain. By Lemma 4, we have that  $0 < V(0) < \lim_{\theta \rightarrow \infty} V(\theta) < \infty$ . Hence,  $\theta^* \rightarrow \infty$

- Let  $\epsilon \in (1, \infty)$  which implies  $\beta \in (0, 1)$ . We consider there cases.

If  $\pi \in (0, \tilde{\pi}_1]$ , by following the same steps above, we can conclude that  $V(\theta)$  has no critical points on the non-negative domain and, as  $\pi \leq \tilde{\pi}_1 < \tilde{\pi}_2$ ,  $\theta^* = 0$ .

If  $\pi \in (\pi_1, \pi_3)$ , by Lemmas 2 and 3, it follows that  $V(\theta)$  has global maximum on the non-negative domain. From the first order condition (16), with some tedious computations, it is possible to obtain

$$\theta^* = \left( \frac{-a_2 \pi k_1 - a_1(1 - \pi)k_2 + (a_1 - a_2)\sqrt{-\pi k_1(1 - \pi)k_2}}{a_2^2 \pi k_1 + a_1^2(1 - \pi)k_2} \right)^{\frac{1}{b}}$$

with  $k_1 = a_1(\tilde{m}_1 - \tilde{n}_1)$  and  $k_2 = a_2(\tilde{m}_2 - \tilde{n}_2)$ .

If  $\pi \in [\pi_3, 1)$ , by following the same steps above, we can conclude that  $V(\theta)$  has no critical points on the non-negative domain and, as  $\tilde{\pi}_2 < \tilde{\pi}_3 \leq \pi$ ,  $\theta^* \rightarrow \infty$ .

The results of the theorem follow easily from the cases above.  $\square$

### A.3 Proof of the Corollaries 1 and 2

We now prove the two corollaries.

*Proof of Corollary 1.* We start by showing that  $\bar{\pi} \in (\tilde{\pi}_1, \tilde{\pi}_3)$ . Consider first the inequality  $\tilde{\pi}_1 < \bar{\pi}$  which is given by

$$\frac{(\tilde{n}_2 - \tilde{m}_2)a_1}{(\tilde{n}_2 - \tilde{m}_2)a_1 + (\tilde{m}_1 - \tilde{n}_1)a_2} < \frac{a_2(\tilde{n}_2 - \tilde{m}_2)(a_1 + 1)^2}{a_2(\tilde{n}_2 - \tilde{m}_2)(a_1 + 1)^2 + a_1(\tilde{m}_1 - \tilde{n}_1)(a_2 + 1)^2}.$$

After some computations this simplifies in  $(a_1 + 1)^2 a_2^2 > (a_2 + 1)^2 a_1^2$  that is equivalent to  $(a_2 - a_1)(a_1 + a_2 + 2a_1 a_2) > 0$ . Since  $a_1, a_2 > 0$  and  $a_2 > a_1$ , by the assumption on the academics' productivities and  $\beta \in (0, 1)$ , we can conclude that  $\tilde{\pi}_1 < \bar{\pi}$ . Consider now the inequality  $\bar{\pi} < \tilde{\pi}_3$  which is given by

$$\frac{a_2(\tilde{n}_2 - \tilde{m}_2)(a_1 + 1)^2}{a_2(\tilde{n}_2 - \tilde{m}_2)(a_1 + 1)^2 + a_1(\tilde{m}_1 - \tilde{n}_1)(a_2 + 1)^2} < \frac{(\tilde{n}_2 - \tilde{m}_2)a_2}{(\tilde{n}_2 - \tilde{m}_2)a_2 + (\tilde{m}_1 - \tilde{n}_1)a_1}.$$

This is equivalent to show that  $(a_2 + 1)^2 > (a_1 + 1)^2$ . By following the same steps above, we can conclude that  $\bar{\pi} < \tilde{\pi}_3$ . Hence, we can conclude that  $\bar{\pi} \in (\tilde{\pi}_1, \tilde{\pi}_3)$ .

Next, by substituting  $\bar{\pi}$  in equation (8) we indeed find that  $\theta^* = 1$ . Hence, we have that  $\theta^* = 1$  when  $\pi = \bar{\pi}$ .

Finally, note that  $\theta^* = \left( \frac{-B + \sqrt{B^2 - 4AC}}{2A} \right)^{\frac{1}{b}}$  by the previous results. Then,  $\theta^* > 1$  implies  $A + B + C > 0$  as  $A > 0$  and  $b < -1$ . As  $A, B$ , and  $C$ , are increasing in  $\pi$ , we conclude that  $\theta^* > 1$  when  $\pi > \bar{\pi}$  and that  $\theta^* < 1$  when  $\pi < \bar{\pi}$ .  $\square$

*Proof of Corollary 2.* To simplify the notation, let's denote the fraction in the brackets of (8) with  $K$ , i.e.,  $K = \frac{-a_2\pi k_1 - a_1(1-\pi)k_2 + (a_1 - a_2)\sqrt{-\pi k_1(1-\pi)k_2}}{a_2^2\pi k_1 + a_1^2(1-\pi)k_2}$ . We can then define the function  $\theta^*(\rho) = K^{\frac{\beta-1}{\rho-\beta}}$  that associates to any  $\rho$  the optimal targets. Consider now its derivative which is

$$\frac{d\theta^*(\rho)}{d\rho} = \frac{\log(K)(1-\beta)K^{\frac{\beta-1}{\rho-\beta}}}{(\beta-\rho)^2},$$

where  $\beta \in (0, 1)$ . For  $K > 1$ , we have  $\theta^* < 1$  and  $\frac{\partial\theta^*(\rho)}{\partial\rho} > 0$ . For  $0 < K < 1$ , we have  $\theta^* > 1$  and  $\frac{\partial\theta^*(\rho)}{\partial\rho} < 0$ .  $\square$

## A.4 Proof of Theorem 3

*Proof of Theorem 3.* The manager maximization problem in (12) can be rewritten as

$$\begin{aligned} \sup_{\theta_1, \theta_2} \quad & \pi [\tilde{m}_1 e_1(\theta_1) + \tilde{n}_1 (E - e_1(\theta_1))] + (1 - \pi) [\tilde{m}_2 e_2(\theta_2) + \tilde{n}_2 (E - e_2(\theta_2))], \\ \text{s.t.} \quad & \theta_1 \in [0, \infty), \\ & \theta_2 \in [0, \infty). \end{aligned}$$

Note that the objective function is separable in the two variables  $\theta_1$  and  $\theta_2$ . Therefore, the manager maximizes the output of the two types of academics independently. Consider first the academics of type 1. Since  $\tilde{m}_1 > \tilde{n}_1$ , as shown in the proof of Lemma 1, and  $e_1(\theta)$  is an increasing function, by Proposition 1, the manager's optimal personalized targets are  $\theta_1^* \rightarrow \infty$ , i.e., academics of type 1 full specialize in research. Consider next the academics of type 2. By following, *mutatis mutandis*, the same steps above, we conclude that the manager's optimal personalized targets are  $\theta_2^* = 0$ , i.e., academics of type 2 full specialize in teaching.  $\square$

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